

Proposal number: GRD1-1999-20007  
Project acronym: HAMSTAD  
Project full title: Determination of liquid water transfer properties  
of porous building materials and development of  
numerical assessment methods.

## **HAMSTAD- WP 2 Modeling**

Carl-Eric Hagentoft  
Building Physics  
Chalmers University of Technology, Sweden

Building Physics, Report R-02:9

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Modeling in the design process</b>	<b>1</b>
<b>3</b>	<b>Physical models</b>	<b>2</b>
3.1	Heat and mass balance . . . . .	4
3.2	Transfer of Heat, Air and Moisture . . . . .	4
3.3	Exterior boundary and climate condition . . . . .	5
3.4	Interior boundary and climate conditions . . . . .	5
<b>4</b>	<b>Mathematical models: Energy and mass balance</b>	<b>5</b>
4.1	Air . . . . .	5
4.2	Moisture . . . . .	6
4.3	Energy . . . . .	6
<b>5</b>	<b>Mathematical models for transport equations - Air</b>	<b>7</b>
<b>6</b>	<b>Mathematical models for transport equations - Heat</b>	<b>7</b>
<b>7</b>	<b>Mathematical models for transport equations - Moisture</b>	<b>8</b>
7.1	General moisture flow equation - Vapor diffusion and liquid transport . . . . .	8
7.2	Simplifications - Vapor diffusion and liquid transport . . . . .	8
7.3	Vapor transport due to convection . . . . .	10
<b>8</b>	<b>Mathematical model - Initial conditions</b>	<b>10</b>
<b>9</b>	<b>Mathematical models</b>	
	<b>- Boundary conditions for 1D cases</b>	<b>10</b>
9.1	Moisture . . . . .	10
9.2	Heat . . . . .	11
9.3	Interior boundaries . . . . .	11
<b>10</b>	<b>Summary of equations for simplified model</b>	<b>12</b>
10.1	Heat flow . . . . .	12
10.2	Moisture flow . . . . .	12
10.3	Energy and mass balance . . . . .	12

# 1 Introduction

This document suggests simplifications in the physical models required to assess the HAM performance of structures.

## 2 Modeling in the design process

The following algorithm is a way of structuring the design process in terms of Building Physics.

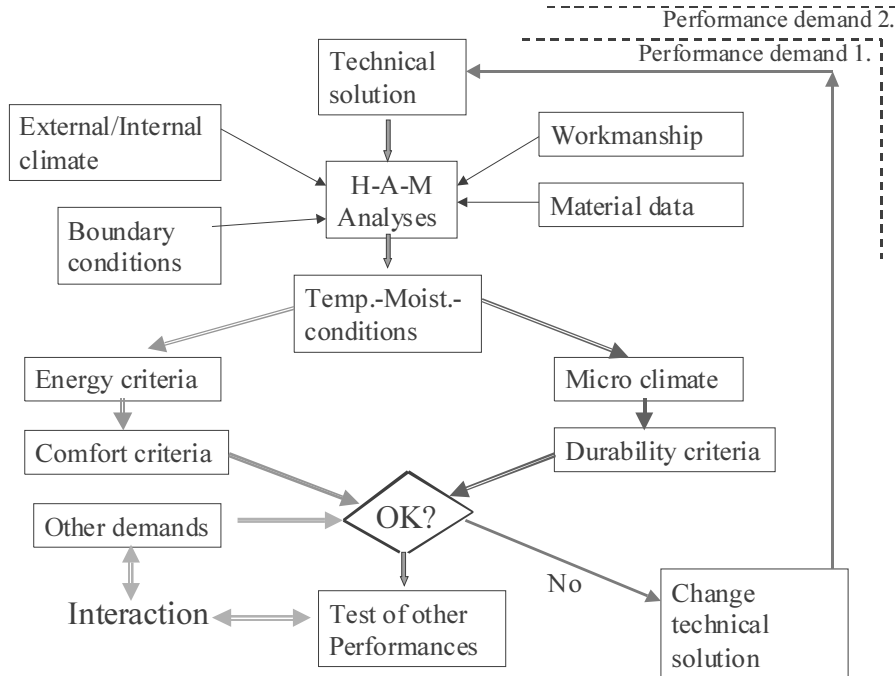


Figure 1: Building Physics Design Algorithm.

The idea with modeling is to predict the performance of structures in advance. To avoid bad technical solutions and enhance the good ones. Important performances deal with:

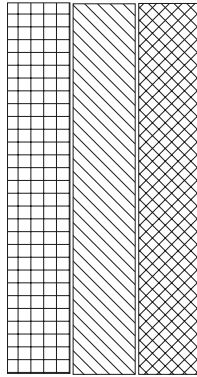
- Indoor air quality
- Thermal comfort
- Energy efficiency
- Durability

In order to determine if these performances are met, accurate calculations of temperatures, moisture conditions, inside and at the surfaces of the structures as well as heat and mass transfer through the structures. These data must then be combined with acceptable models or/and requirements for the above listed performances, see (Hagentoft, 1998).

### 3 Physical models

The simple Glaser model has been used frequently in many countries to assess moisture safety of structures. This one-dimensional model has, as everyone knows, a lot of limitations. For instance, it cannot handle either the capacity of heat nor moisture. Neither can it handle air transfer through structures.

In order to improve the modeling tools for designers, a reasonable first step is to be able to treat *layered one-dimensional structures*, based on a sound physical model. There are limitation in the available models, in particular for moisture transfer. Nevertheless, dramatically improved predictions can be obtained. The multi-layered structures are part of the building envelope. The contact between the layers can either be direct or through a ventilated or non-ventilated air gap. However, ventilated air gaps are not accounted for here.



*Figure 2: Considered type of layered one-dimensional structure.*

The following modeling components have to be accounted for:

- Heat and mass balance
- Transfer of heat, air and moisture
- Exterior boundary and climate condition
- Interior boundary and climate conditions

These components will be discussed in the following sections.

The physical models concern buildings on the surface of earth, in Europe. From this follows a number natural limitations.

- Constant atmospheric pressure (minor fluctuations, slow in time)
- Naturally occuring weather conditions or ground conditions

- Temperature is normally in the range of say  $-30^{\circ}\text{C}$   $+80^{\circ}\text{C}$ , in the simplified model temperatures are assumed to be above  $0^{\circ}\text{C}$ , formation of ice is not considered.
- Simplifications for driving rain
- Regular building materials
- Constant geometry and dimensions of structural components
- Sorption isotherm is temperature independent
- No chemical reaction within structures
- No gravity effects (only coupled to run off at surfaces due to water saturation of surface materials)
- No drainage between material layers
- No ageing
- No hysteresis
- Materials is never more than capillary moisture saturated
- No ventilated cavities in structure

There are a number of equations giving relations between different moisture state. We can use  $p$ ,  $v$  or  $\phi$  to characterize the moisture state of the humid air.

The relation between the partial pressure  $p$  and the humidity by volume  $v$  is according to the General gas law:

$$p = v \cdot \frac{RT}{M_w} \quad (1)$$

The general gas constant  $R$  has the value:  $8.314 \text{ J}/(\text{mol},\text{K})$ . The molar weight of water vapor  $M_w$  is  $0.018 \text{ kg}/\text{mol}$ . The temperature,  $T$ , is given in Kelvin. In particular the saturation vapor pressure  $p_s(T)$  and the corresponding humidity by volume at saturation  $v_s(T)$  are related as:

$$p_s(T) = v_s(T) \cdot \frac{RT}{M_w} \quad (2)$$

Here, the temperature is in Kelvin. The relative humidity  $\phi$  is defined by:

$$\phi = \frac{p}{p_s} = \frac{v}{v_s} \quad (3)$$

Consider water vapor (gas phase) and capillary condensed water in close contact within the pores. The two phases are in equilibrium when the pore water pressure  $P_\ell$  and the relative humidity  $\phi$  satisfies Kelvin's law:

$$P_\ell = p_s(T) + \frac{RT\rho_w}{M_w} \ln(\phi) \quad (4)$$

Here,  $\rho_w \simeq 1000 \text{ kg/m}^3$  is the density of liquid water. Water vapor will condense on the liquid phase, when the relative humidity is higher than the equilibrium value, and water will evaporate for a lower  $\phi$ . Since the first term often is considerable smaller than the second one, the following approximation can be used:

$$P_\ell = \frac{RT\rho_w}{M_w} \ln(\phi) \quad (5)$$

The pressure difference  $P_g - P_\ell$  is denoted the suction pressure,  $P_{suc}$ :

$$P_g - P_\ell = P_{suc} \quad (6)$$

Here,  $P_g$ , refers to the pressure of the surrounding gas (air). We also have the Thomson's formula, which relates pore radius with relative humidity:

$$\phi = e^{-\frac{2\sigma}{r} \frac{M_w}{\rho_w RT}} = e^{-P_{suc} \cdot \frac{M_w}{\rho_w RT}} \quad (7)$$

Here,  $\sigma$  ( $73 \cdot 10^{-3} \text{ N/m}$  at  $20 \text{ }^\circ\text{C}$ ) is a surface tension coefficient.

### 3.1 Heat and mass balance

There are two important conservation requirements:

- Conservation of energy
- Conservation of mass (air and moisture)

These requirements must always be fulfilled.

Considering the restriction of constant pressure the heat storage can be handled by the use of heat capacities (for constant pressure) for both air and building materials. In addition to this, changes in latent heat (phase changes between liquid and gas) must be considered.

With a constant air pressure the net inflow of air (volume flow) to a control volume is zero.

The net inflow of moisture (both in liquid and vapor phase) to a control volume is balanced by the storage of moisture in materials.

### 3.2 Transfer of Heat, Air and Moisture

Air flows are caused by minor fluctuations (in comparison with atmospheric pressure) in the air pressure caused by:

- Wind
- Stack effect (temperature differences)
- Mechanical ventilation

The air is flowing through permeable building materials in the layered structure.

Transfer of heat is caused by:

- Conduction
- Convection (air and water liquid flow, heat carried by the heat capacity and latent heat)
- Radiation (long wave and solar radiation)

Moisture flows are caused by:

- Water vapor diffusion
- Capillary actions (moving liquid water)
- Convection (of water vapor)

### 3.3 Exterior boundary and climate condition

The following components of the exterior climate must be accounted for:

- Exterior air temperature and relative humidity
- Solar and long wave radiation
- Driving rain
- Wind pressure

### 3.4 Interior boundary and climate conditions

The following components of the exterior climate must be accounted for:

- Interior air temperature and relative humidity
- Interior pressure conditions

## 4 Mathematical models: Energy and mass balance

### 4.1 Air

Air mass flow rate is denoted  $\mathbf{m}_a$ (kg/m<sup>2</sup>s). Conservation of mass gives:

$$-\nabla \cdot \mathbf{m}_a = 0 \quad (8)$$

We have neglected variations in air density. With  $\mathbf{m}_a = \mathbf{r}_a \cdot \rho_a$ , where  $\mathbf{r}_a$  is air flow rate (m<sup>3</sup>/m<sup>2</sup>s), neglecting variations in air density with temperature gives:

$$-\nabla \cdot \mathbf{r}_a = 0 \quad (9)$$

We have, since constant atmospheric pressure is assumed, constant air density.

## 4.2 Moisture

Moisture can exist in three different phases; vapor, liquid and solid (ice). In the simplified model focused on here, ice is neglected.

Moisture is transported in either vapor or liquid phase. The moisture flow rate  $\mathbf{g}$  (kg/m<sup>2</sup>s) becomes:

$$\mathbf{g} = \mathbf{g}_v + \mathbf{g}_l \quad \mathbf{g}_v = \mathbf{g}_{v,d} + \mathbf{g}_{v,c} \quad (10)$$

Here, the transport due to vapor has been divided into one due to diffusion and another one due to convection. The source term for the amount of evaporating moisture, from liquid, is denoted  $G_v$  (kg/m<sup>3</sup>s). The corresponding term for the amount of condensed vapor is denoted  $G_l$  (kg/m<sup>3</sup>s). The following balance must be satisfied:

$$G_v + G_l = 0 \quad (11)$$

This means that the changes in phase do not create any net amount of water.

The moisture balance equation for a porous media reads:

$$-\nabla \cdot \mathbf{g} = \frac{\partial w}{\partial t} \quad w = w_v + w_l \quad (12)$$

Here,  $\mathbf{g}$  is the moisture flow (kg/m<sup>2</sup>s), and  $w$  the moisture content (both phases). The subscripts refer to vapor and liquid. Using the fraction of porosity  $\psi_p$  we have the following relation:

$$w_v = \psi_p \cdot v \quad (13)$$

The following mass balances must be satisfied:

$$-\nabla \cdot \mathbf{g}_v + G_v = \frac{\partial w_v}{\partial t} \quad (14)$$

$$-\nabla \cdot \mathbf{g}_l + G_l = \frac{\partial w_l}{\partial t} \quad (15)$$

## 4.3 Energy

The heat flow rate can be divided into one due to conduction and one due to convection.

$$\mathbf{q} = \mathbf{q}_{\text{cond}} + \mathbf{q}_{\text{conv}} \quad (16)$$

The first term gives the heat flow rate (W/m<sup>2</sup>), due to conduction. It represents all heat flow not associated with macroscopic movements of gases or liquids, i.e interior microscopic conduction, convection and long wave radiation. The second term represents convective flows both carrying heat due to the heat capacity of air, vapor and liquid as well as latent heat.



The energy balance equation reads:

$$-\nabla \cdot (\mathbf{q}_{\text{cond}} + \mathbf{q}_{\text{conv}}) = c_0 \rho_0 \frac{\partial T}{\partial t} + c_{p,v} w_v \frac{\partial T}{\partial t} + c_l w_l \frac{\partial T}{\partial t} + \ell_{lv} \frac{\partial w_v}{\partial t} \quad (17)$$

The dry density of the material is  $\rho_0$  and the heat capacity (including dry air in pores) is  $c_0$ . We can define a lumped capacity term:

$$c = c_0 + \frac{1}{\rho_0} (c_{p,v} w_v + c_l w_l) \quad (18)$$

Formula (17) rewritten gives:

$$-\nabla \cdot (\mathbf{q}_{\text{cond}} + \mathbf{q}_{\text{conv}}) = c \rho_0 \frac{\partial T}{\partial t} + \ell_{lv} \frac{\partial w_v}{\partial t} \quad (19)$$

The equation can be read as a balance between the net inflow of heat to a volume and the increase of energy related to heat capacities and temperature changes, and the increase of the amount of water vapor, within the volume.

## 5 Mathematical models for transport equations - Air

For isotropic materials we have the Darcy equation for low air flow rates (laminar flow):

$$\mathbf{r}_a = -\frac{k_a}{\eta} \nabla P_a \quad (20)$$

For one-dimensional cases, the air flow rate must be constant through the structure, as a consequence of mass conservation (8). A general expression for the air flow rate is determined by the air pressure difference over the structure according to.

$$r_a = C \cdot \Delta P_a^\kappa \quad (21)$$

Here,  $\kappa$  is equal to 1 for laminar flow and 0.5 for fully turbulent flow.

## 6 Mathematical models for transport equations - Heat

The transport equation for heat conduction for isotropic materials reads:

$$\mathbf{q}_{\text{cond}} = -\lambda \nabla T \quad (22)$$

Choosing  $T=0$  °C (and liquid water) as the reference for the energy (enthalpy) flows, the convective heat flow reads:

$$\mathbf{q}_{\text{conv}} = \mathbf{r}_a \rho_a c_{p,a} \cdot T + \mathbf{g}_l c_{p,l} \cdot T + \mathbf{g}_v \cdot (\ell_{lv} + c_{p,v} \cdot T) \quad (23)$$

Here, the  $c_p$  (J/kgK) refers to the specific heat at constant atmospheric conditions, and  $\ell_{lv}$  (J/kg) to the latent heat of evaporation.

## 7 Mathematical models for transport equations - Moisture

The diffusion and liquid transport process is discussed in Section 7.1-2, while the convective part is stated in Section 7.3.

### 7.1 General moisture flow equation - Vapor diffusion and liquid transport

In non-isothermal moisture flow it is necessary to account for two independent state variables. One possible combination is the humidity by volume and the temperature. The moisture flow is then given by:

$$\mathbf{g}_{v,d} + \mathbf{g}_l = -D_v^T \nabla v - D_T^v \nabla T \quad (24)$$

Here, the subscript of the flow coefficient  $D$  denotes the corresponding potential, and the superscript the second potential that is used in the formula. The two flow coefficients are functions of the two state variables:

$$D_v^T = D_v^T(v, T) \quad D_T^v = D_T^v(v, T) \quad (25)$$

In a similar way the humidity by volume and the moisture content can be chosen as potentials.

$$\mathbf{g}_{v,d} + \mathbf{g}_l = -D_v^w \nabla v - D_w^v \nabla w \quad (26)$$

, with

$$D_v^w = D_v^w(v, w) \quad D_w^v = D_w^v(v, w) \quad (27)$$

To determine the flow coefficients a lot of measurements are required. Both coefficients are functions of two variables. Unfortunately such sets of material data are not available today. For non-isothermal flows we have to introduce simplifications.

### 7.2 Simplifications - Vapor diffusion and liquid transport

At *isothermal conditions*, it is sufficient to use one potential for moisture transport. Nevertheless, it can be useful to divide the moisture transport in to one vapor and one liquid part. This is of course only an approximation since the combined processes can strictly not be divided. An approximation is then given by:

$$\mathbf{g} = \mathbf{g}_{v,d} + \mathbf{g}_l \quad (28)$$

$$\mathbf{g}_{v,d} = -\delta_p(w, T) \nabla p \quad (29)$$

$$\mathbf{g}_l = K(w, T) \nabla P_{suc} \quad K = \frac{k_\ell(w) \rho_w}{\eta(T)} \quad (30)$$

Here, the first term accounts for vapor diffusion flow only. The vapor permeability will be rather constant for dry materials, when the pore system is open for diffusion. The diffusion part will diminish and go down to zero when the pore system is saturated with moisture. The diffusion coefficient will also depend on the temperature. The second part takes care of the liquid part, it is given by a Darcy formulation and the suction pressure gradient in the materials. The term  $k_\ell$  ( $\text{m}^2$ ) is the permeability and  $\eta$  is the dynamic viscosity ( $\text{kg/ms}$ ).

The vapor flow part will be strongly influenced by a temperature gradient at *non-isothermal* conditions, while the liquid part is only weakly influenced. Formulas (28-30) can be used as an approximation for these cases as well.

The state variable equations (1) to (7), together with the sorption isotherm can be used for transformations to other choices of variables or ‘potentials’. The slope of the sorption isotherm (neglecting temperature dependence) is denoted:

$$\xi = \frac{dw}{d\phi} \quad (31)$$

Another relation that will be used is the Clausius-Clapeyrons relation:

$$\frac{dp_s(T)}{dT} = \frac{M_w \ell_{lv}}{RT^2} \cdot p_s \quad (32)$$

To illustrate the transformation, see appendix for details, we have for instances using  $p$  and  $T$  as state variables:

$$\mathbf{g}_{\mathbf{v},\mathbf{d}} = -\delta_p \nabla p \quad (33)$$

$$\mathbf{g}_{\mathbf{l}} = -K \cdot \frac{R\rho_w}{M_w} \left\{ \left( \ln(\phi) - \frac{T}{p_s} \frac{dp_s(T)}{dT} \right) \nabla T + \frac{T}{p} \nabla p \right\} \quad (34)$$

Using  $p$  and  $w$  instead gives:

$$\mathbf{g}_{\mathbf{v},\mathbf{d}} = -\delta_p \nabla p \quad (35)$$

$$\mathbf{g}_{\mathbf{l}} = -K \cdot \frac{R\rho_w}{M_w} \frac{T}{p} \nabla p - D_w \nabla w \quad (36)$$

were

$$D_w = -K \cdot \frac{\partial P_{suc}}{\partial w} \text{ or } D_w = K \cdot \frac{RT\rho_w}{M_w} \frac{1}{\phi\xi} \quad (37)$$

The first expression, in (37), is normally used in the over hygroscopic region, and the second one in the hygroscopic region.

From (Andersson, 1985) we also have, using  $w$  and  $T$  as potentials:

$$\mathbf{g}_{\mathbf{v},\mathbf{d}} = -\delta_p p_s \frac{1}{\xi} \nabla w - \delta_p p_s \left( \left. \frac{\partial \phi}{\partial T} \right|_{w,fix} + \phi \frac{M_w \ell_{lv}}{RT^2} \right) \nabla T \quad (38)$$

$$\mathbf{g}_{\mathbf{l}} = K \cdot P_{suc} \cdot \gamma \nabla T + K \cdot \frac{\partial P_{suc}}{\partial w} \nabla w \quad (39)$$

were  $\gamma$  is given by the following expression containing the surface tension coefficient.

$$\gamma = \frac{1}{\sigma} \frac{d\sigma}{dT} \quad (40)$$

Using  $P_{suc}$  and  $T$  as potentials we get:

$$\mathbf{g}_{v,d} = \delta_p \left( \frac{pM_w}{\rho_w RT} \right) \nabla P_{suc} - \delta_p p \left( \frac{M_w \ell v}{RT^2} - P_{suc} \frac{M_w \gamma}{\rho_w RT} + P_{suc} \frac{M_w}{\rho_w RT^2} \right) \nabla T \quad (41)$$

$$\mathbf{g}_l = K \nabla P_{suc} \quad (42)$$

### 7.3 Vapor transport due to convection

Vapor is carried by the air flow:

$$\mathbf{g}_{v,c} = \mathbf{r}_a \cdot v \quad (43)$$

## 8 Mathematical model - Initial conditions

It is necessary to know the initial moisture content and temperature for time dependent processes.

$$w(t=0) = f_1(x) \quad T(t=0) = f_2(x) \quad (44)$$

## 9 Mathematical models - Boundary conditions for 1D cases

In the following, the normal,  $\hat{n}$ , is pointing into the material.

### 9.1 Moisture

The total moisture flux (vapor flux)  $g_{v,n}$  (kg/m<sup>2</sup>) to the surface of the structure must be handled.

Air flowing into the structure:

$$g_{v,n} = \beta_p \cdot (p_a - p_{surf}) + \mathbf{r}_a \cdot \hat{n} \cdot v_a \quad (45)$$

Air flowing out from the structure:

$$g_{v,n} = \beta_p \cdot (p_a - p_{surf}) + \mathbf{r}_a \cdot \hat{n} \cdot v_{surf} \quad (46)$$

Here,  $p_a$  (Pa) is the ambient partial vapor pressure.

Suction of water from driving rain can take place for capillary active surfaces. The liquid inflow  $g_{l,n}$  is max of:

$$K \cdot \frac{\partial P_{suc}}{\partial n} \quad (47)$$

and a prescribed liquid flux:

$$g_{liquid\ flux} \quad (48)$$

The moisture content at surfaces is limited to  $w_{cap}$ .

## 9.2 Heat

The heat flux  $q_n$  (W/m<sup>2</sup>) to the surface must also be handled.

Air (and liquid moisture) flowing into the structure:

$$q_n = \alpha_e \cdot (T^{eq} - T_{surf}) + \mathbf{r}_a \cdot \hat{n} \rho_a c_{p,a+v} \cdot T_a + g_{l,n} c_{p,l} \cdot T_a + g_{v,n} \cdot \ell_{lv} \quad (49)$$

Air flowing out (and no liquid moisture inflow) from the structure:

$$q_n = \alpha_e \cdot (T^{eq} - T_{surf}) + \mathbf{r}_a \cdot \hat{n} \rho_a c_{p,a+v} \cdot T_{surf} + g_{v,n} \cdot \ell_{lv} \quad (50)$$

Here, an effective heat transfer coefficient  $\alpha_e$  has been introduced:

$$\alpha_e = \alpha_c + \alpha_r \quad (51)$$

The convective heat transfer coefficient  $\alpha_c$  depends on air flow velocities and temperature differences between the surface and the air. The radiative heat transfer coefficient  $\alpha_r$  depends on the temperature of the surroundings (sky temperatures) and the temperature of the surface as well as the emissivity of the surface. The equivalent temperature is:

$$T^{eq} = T_a + \frac{1}{\alpha_e} (I_{sol} \cdot \alpha_{sol} + (T^r - T_a) \cdot \alpha_r) \quad (52)$$

Here, the solar radiation component, parallel to the normal of the structure, including both diffuse and direct radiation (global radiation), is denoted by  $I_{sol}$  (W/m<sup>2</sup>). The ambient air temperature is  $T_a$ . The temperature  $T^r$  depends on the surrounding surfaces and the atmosphere that has a long wave radiation exchange with the outer surface. The solar radiation terms must be obtained from metrological registrations, taken into account possible shading due to local conditions. In a similar way the radiation temperature,  $T^r$ , can be calculated using the cloudiness, the air temperature, the inclination of the building envelope component and the local surroundings.

For internal surfaces we simply have  $T^{eq} = T_a$ .

## 9.3 Interior boundaries

In case of discontinuity in materials or two materials in imperfect contact the vapor flow is given by:

$$g_v = \frac{\Delta p}{Z_{p,c}} \quad (53)$$

Here,  $Z_{p,c}$  (m/s) is a vapor contact resistance, it can for instance also account for the resistance of a very thin vapor retarder placed between two layers.

In case of discontinuity in materials or two materials in imperfect contact the liquid flow is given by:

$$g_l = c \Delta P_{suc} \quad (54)$$

Here,  $c$  ( $\text{kg}/\text{m}^2\text{sPa}$ ) is a liquid transfer coefficient. For the case of contact between a capillary active and a capillary non active material,  $c$  becomes zero. The coefficient can also account for very thin capillary breaking materials placed between two layers.

In case of discontinuity in materials or two materials in imperfect contact, the heat flow is given by:

$$q = \frac{\Delta T}{R_c} \quad (55)$$

Here,  $R_c$  ( $\text{m}^2\text{K}/\text{W}$ ) is a thermal resistance. The coefficient can also account the thermal resistance cross a non-ventilated air layers, i.e accounting for the resulting thermal resistance cross the layer due to radiation, conduction and interior convection.

## 10 Summary of equations for simplified model

### 10.1 Heat flow

The heat flow due to conduction reads:

$$\mathbf{q}_{\text{cond}} = -\lambda(\mathbf{w})\nabla T \quad (56)$$

Here, we have neglected the temperature dependence for the thermal conductivity, due to the assumed limited temperature span. Neglecting heat transfer ‘carried’ by the heat capacity of vapor and liquid water, and to liquid flow or vapor diffusion, the convective heat flow (23) can be approximated as:

$$\mathbf{q}_{\text{conv}} = \mathbf{r}_a \rho_a c_{p,a} \cdot T + \mathbf{g}_v \cdot \ell_{lv} \quad (57)$$

### 10.2 Moisture flow

The assumed approximation for the moisture flow is given by:

$$\mathbf{g} = \mathbf{g}_v + \mathbf{g}_l \quad (58)$$

$$\mathbf{g}_v = -\delta_p(\mathbf{w})\nabla p + \mathbf{r}_a \cdot v \quad (59)$$

$$\mathbf{g}_l = K(\mathbf{w}, \bar{T})\nabla P_{suc} \quad \bar{\eta} = \eta(\bar{T}) \quad (60)$$

### 10.3 Energy and mass balance

Mass balance is expressed by:

$$-\nabla \cdot \mathbf{g} = \frac{\partial \mathbf{w}}{\partial t} \quad (61)$$

For a one-dimensional case we have:

$$\frac{\partial}{\partial x} \left( \delta_p \frac{\partial p}{\partial x} \right) - r_a \cdot \frac{\partial v}{\partial x} - \frac{\partial}{\partial x} \left( K \frac{\partial P_{suc}}{\partial x} \right) = \frac{\partial w}{\partial t} \quad (62)$$

The energy balance is given by (19). Neglecting the dependence of the vapor content in the material pores, see (17-18) we get:

$$-\nabla \cdot (\mathbf{q}_{\text{cond}} + \mathbf{q}_{\text{conv}}) = c\rho_0 \frac{\partial T}{\partial t} \quad (63)$$

$$c = c_0 + \frac{1}{\rho_0} c_l w_l \quad (64)$$

For one-dimensional case the heat balance equation becomes:

$$\frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - r_a \rho_a c_{p,a} \cdot \frac{\partial T}{\partial x} + \ell_{lv} \frac{\partial}{\partial x} \left( \delta_p \frac{\partial p}{\partial x} \right) - r_a \ell_{lv} \cdot \frac{\partial v}{\partial x} = c\rho_0 \frac{\partial T}{\partial t} \quad (65)$$

Formulas for boundary, initial and contact conditions are stated in Sections 8-9.

# Nomenclature

$c$	Liquid transfer coefficient for contact surfaces ( $\text{kg}/\text{m}^2\text{sPa}$ )
$c_p$	Specific heat capacity at constant pressure ( $\text{J}/\text{kgK}$ )
$D_w$	Moisture diffusivity ( $\text{m}^2/\text{s}$ )
$g$	Density of moisture flow rate ( $\text{kg}/\text{m}^2\text{s}$ )
$G$	Moisture source term ( $\text{kg}/\text{m}^3\text{s}$ )
$k_a, k_\ell$	Permeability ( $\text{m}^2$ )
$\ell$	Latent heat associated with phase change ( $\text{J}/\text{kg}$ )
$\ell_{lv}$	Latent heat of evaporation ( $\text{J}/\text{kg}$ )
$M_w$	Molar weight of water ( $\text{kg}/\text{mol}$ )
$p$	Partial vapor pressure (Pa)
$P$	Pressure (Pa)
$P_{suc}$	Suction pressure (Pa)
$q$	Density of heat flow rate ( $\text{W}/\text{m}^2$ )
$r_a$	Density of air flow rate ( $\text{m}^3/\text{m}^2\text{s}$ )
$R$	Gas constant ( $\text{J}/\text{mol K}$ )
$R$	Thermal resistance ( $\text{m}^2\text{K}/\text{W}$ )
$t$	Time (s)
$T$	Temperature ( $^\circ\text{C}$ , K)
$v$	Humidity by volume ( $\text{kg}/\text{m}^3$ )
$w$	Moisture content mass by volume ( $\text{kg}/\text{m}^3$ )
$Z_p$	Moisture resistance of film ( $\text{m}/\text{s}$ ) or ( $\text{m}^2\text{s Pa}/\text{kg}$ )
$\alpha$	Surface heat transfer coefficient ( $\text{W}/\text{m}^2\text{K}$ )
$\beta_p$	Surface coefficient of water vapor transfer ( $\text{s}/\text{m}$ )
$\gamma$	Temperature dependence of the surface tension coefficient ( $1/\text{K}$ )
$\delta_p$	Moisture permeability (s) or ( $\text{kg}/\text{m s Pa}$ )
$\epsilon$	Emissivity (-)
$\eta$	Dynamic viscosity ( $\text{kg}/\text{ms}$ )
$\kappa$	Pressure coefficient (-)
$\lambda$	Thermal conductivity ( $\text{W}/\text{mK}$ )
$\rho$	Density ( $\text{kg}/\text{m}^3$ )
$\sigma$	Surface tension coefficient ( $\text{N}/\text{m}$ )
$\phi$	Relative humidity (-)
$\xi$	Moisture differential capacity ( $\text{kg}/\text{m}^3$ )
$\psi_p$	Fraction of porosity (-)

Bold face characters indicate that it is a flow vector. When an upper bar is placed on top of a variable, the average value is pointed out.

Index:

a	air or ambient
c	convective, contact
d	diffusive
f	fluid



i	ice (solid phase)
i,e	interior or exterior
l	liquid
n	normal direction
r	radiation
s	saturation
suc	suction
surf	surface
v	vapor

## References

1. Andersson A.-C., 1985. Verification of calculation methods for moisture transport in porous building materials. Report D6:1985, Swedish Council for Building Research.
2. Hagentoft C-E. 1998, Final report, IEA/Annex 24, Task 5. The Impact of Heat and Moisture transport on Energy Demand and Durability, Chalmers University of Technology, Dept. of Building Physics, P-98:2.

## Appendix: Derivations

### State variables $p$ and $T$

The gradient in liquid pressure must be rewritten so it is expressed in terms of  $p$  and  $T$

$$-\nabla P_{suc} = \nabla P_\ell = \frac{R\rho_w}{M_w} \left\{ \nabla T \ln(\phi) + \frac{T}{\phi} \nabla \phi \right\} \quad (66)$$

The gradient of the relative humidity becomes:

$$\nabla \phi = \nabla \left( \frac{p}{p_s} \right) = \frac{1}{p_s} \nabla p - \frac{\phi}{p_s} \frac{dp_s}{dT} \nabla T \quad (67)$$

Inserted:

$$-\nabla P_{suc} = \frac{R\rho_w}{M_w} \left\{ \nabla T \ln(\phi) + \frac{T}{\phi} \left( \frac{1}{p_s} \nabla p - \frac{\phi}{p_s} \frac{dp_s}{dT} \nabla T \right) \right\} \quad (68)$$

Sorting gives:

$$-\nabla P_{suc} = \frac{R\rho_w}{M_w} \left\{ \left( \ln(\phi) - \frac{T}{\phi} \frac{\phi}{p_s} \frac{dp_s}{dT} \right) \nabla T + \frac{T}{\phi} \left( \frac{1}{p_s} \nabla p \right) \right\} \quad (69)$$

$$-\nabla P_{suc} = \frac{R\rho_w}{M_w} \left\{ \left( \ln(\phi) - \frac{T}{p_s} \frac{dp_s}{dT} \right) \nabla T + \frac{T}{p} \nabla p \right\} \quad (70)$$

An alternative way of deriving this is from:

$$\nabla P_{suc} = \left. \frac{\partial P_{suc}}{\partial p} \right|_{T \text{ fix}} \cdot \nabla p + \left. \frac{\partial P_{suc}}{\partial T} \right|_{p \text{ fix}} \cdot \nabla T \quad (71)$$

The first term gives:

$$-\frac{\partial P_{suc}}{\partial p} = -\frac{\partial P_{suc}}{\partial \phi} \frac{\partial \phi}{\partial p} = \frac{RT\rho_w}{M_w} \frac{1}{\phi} \frac{\partial \phi}{\partial p} = \frac{RT\rho_w}{M_w} \frac{1}{\phi p_s} = \frac{R\rho_w}{M_w} \frac{T}{p} \quad (72)$$

The second term gives:

$$-\frac{\partial P_{suc}}{\partial T} = \frac{R\rho_w}{M_w} \left( \ln(\phi) + \frac{\partial \phi}{\partial T} \frac{T}{\phi} \right) \quad (73)$$

$$\frac{\partial \phi}{\partial T} = \frac{\partial(p/p_s)}{\partial T} = \frac{1}{p_s^2} \left( \frac{\partial p}{\partial T} p_s - p \frac{\partial p_s}{\partial T} \right) \quad (74)$$

with the first derivative equal to zero, since  $p$  is considered constant, we have:

$$-\frac{p}{p_s^2} \frac{\partial p_s}{\partial T} \quad (75)$$

Together we get:

$$-\frac{\partial P_{suc}}{\partial T} = \frac{R\rho_w}{M_w} \left( \ln(\phi) - T \frac{p}{p_s^2} \frac{\partial p_s}{\partial T} \frac{1}{\phi} \right) = \frac{R\rho_w}{M_w} \left( \ln(\phi) - \frac{T}{p_s} \frac{\partial p_s}{\partial T} \right) \quad (76)$$

Combining (71-72) and (76) the expression will be equal to (70).

## State variables $p$ and $w$

The gradient in liquid pressure must be rewritten so it is expressed in terms of  $p$  and  $w$ . With the method according to (72):

$$\nabla P_{suc} = \left. \frac{\partial P_{suc}}{\partial p} \right|_{w \text{ fix}} \cdot \nabla p + \left. \frac{\partial P_{suc}}{\partial w} \right|_{p \text{ fix}} \cdot \nabla w \quad (77)$$

The first term gives:

$$-\frac{\partial P_{suc}}{\partial p} = \frac{\partial P_\ell}{\partial \phi} \frac{\partial \phi}{\partial p} \quad (78)$$

$$-\frac{\partial P_{suc}}{\partial p} = \frac{RT\rho_w}{M_w} \frac{1}{\phi} \frac{1}{p_s} = \frac{RT\rho_w}{M_w} \frac{1}{p} \quad (79)$$

The second term:

$$\frac{\partial P_\ell}{\partial w} = \frac{\partial P_\ell}{\partial \phi} \frac{\partial \phi}{\partial w} = \frac{RT\rho_w}{M_w} \frac{1}{\phi \xi} \quad (80)$$